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Different Ways of Expressing Relative Frequencies Numerically and the Struggle of Converting Them

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1 Abstract

Relative frequencies can be represented, e.g., in percent (e.g., 25%), in common fractions (e.g., 1/4), in decimal fractions (e.g., 0.25), in natural frequencies (e.g., 1 out of 4), in the notation, e.g., "every fourth" (we call it the "notation with every"), or in odds (e.g., 1 to 3) (Gigerenzer & Hoffrage, 1995; Krauss et al., 2020). At first, these different representations seem cognitively easy to understand, but how do students really perform when, e.g., their mutual conversion is required? In an empirical study, N=79 German students from grade 6 and 7 participated. Every student had to answer four questions. In the first (closed) item (e.g., "What does 40% mean?"), the students had to decide which of seven given possible conversions were right or which were wrong (e.g., A: "4 out of 10", B: "every fortieth", C: "2/5", D: "one fortieth", etc.). The following three (open) items each required students to actively convert one concrete notation of a relative frequency into others (e.g., "Please convert 'every fourth' into the according percentage and natural frequency."). Results show that students strongly struggle when converting one notation into another. While students in average could only identify 61% of the given alternatives in the first item correctly, the performance with the three open items was even worse: They actively converted only 25% correctly. Furthermore, we were able to identify typically errors: A first noticeable mistake was that the students did not recognize the difference between "out of" in natural frequencies and "to" in odds (e.g., they wrote 3 out of 4 = 3 to 4). A second conspicuous error occurred in the spelling with "every". The students often used numbers they saw (e.g., four in "every fourth") in their answers, although this was not the correct conversion (e.g., every fourth = 4 % or 40 %).Since students struggle with the conversion of relative frequency notations, an implementation of this competence into school curricula considering the typical errors is needed. We propose the introduction to the different ways of expressing relative frequencies in a systematic approach oriented towards the basic concepts of natural frequencies (Wiesner et al., in press). In this approach, the explicit new conversions to be learned are reduced by always choosing a path via the natural frequencies. Thus, instead of 30 conversions, only 3 reciprocal conversion principles must be taught.

2 Theoretical Background

- Statistical statements often contain relative frequencies (Krauss et al., 2020)
- There are different expressions with different basic concepts for relative frequencies:

expression	example	(possible) basic concept
percent	25 %	„out of hundred"
decimal fraction	0.25	table of positional
common fraction	$\frac{1}{4}$	
natural frequencie	1 out of 4	
notation with „every"	every fourth	
odds	1:3 („1 to 3")	

- The use of natural frequencies in tasks, for example in exercices on Bayesian contexts, has proven to be helpful (Binder et al., 2020; McDowell & Jacobs, 2017).

3 Research Questions

These six different representations seem cognitively easy to understand, but can students convert the representations into each other without explicit explanation?

4 Design

- *N* = 79 German students from grade 6 and 7
- One test (in three versions) without prior instrcutions
- Each version consisted of one closed item and three open items

References

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5 Instruments & Results

	task	true	false	performance
1) What is 40%?				
	expression			
a)	a chance of „4 to 10"	□	□	43 %
b)	2/5	□	□	48 %
c)	one fortieth	□	□	54 %
d)	four hundred out of thousand	□	□	66 %
e)	4 out of 10	□	□	68 %
f)	every fortieth	□	□	71 %
g)	0.4	□	□	78 %
2) Please convert „every fifth"				
a)	... into percent: ____ %			18 %
b)	... into a common fraction: ___			25 %
3) Please convert „4 out of 6"				
a)	„12 out of ___"			38 %
b)	„a chance of ____ to _____"			0 %
4) Fill in the gap				
Patrick hit two out of ten free throws in basketball. He hit every ___ throw.				47 %

Students' performance roughly corresponded to the 50 % guessing probability.

Students' performance was above the 50 % guessing probability

Students **are not able** to understand these expressions without systematic explanation

Students often used numbers they saw (e.g., four in "every fourth") in their answers, although this was not the correct conversion (e.g., every fourth= 4 % or 40 %).

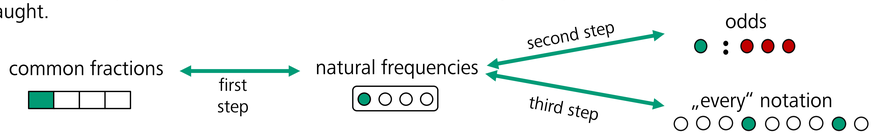
The students did not recognize the difference between "out of" in natural frequencies and "to" in odds.

This question with a context had the highest performance of all half-opend questions.

6 Discussion & Conclusions for school

- An implementation of this competence into school curricula considering the typically errors is needed.
- Proposal of an introduction to the different ways of expressing relative frequencies in a systematic approach with the help of an approach oriented towards the basic concepts (Wiesner et al., in press).

→ In this approach, the explicit new conversions to be learned are reduced by always choosing a path via the natural frequencies. Thus, instead of 30 conversions, only 3 reciprocal conversion principles must be taught.



Additional Information

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